Univerzitet u Zenici
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Parcijalni ispit iz predmeta Matematika I

GRUPA A

1. Dokazati matematičkom indukcijom tvrdnju:

   $$\sin x + \sin 3x + \sin 5x + ... + \sin(2n-1)x = \frac{\sin^2 nx}{\sin x} \ (n \in \square).$$

2. Izračunati sve vrijednosti korjena $\sqrt[3]{z}$, ako je $z = \left(1+i\sqrt{3}\right)(1+i)\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)$.

3. Izračunati sve visine u trouglu ABC ako je $\overrightarrow{AB} = 2\vec{p} - \vec{q}, \overrightarrow{BC} = 3\vec{q}, |\vec{p}| = 2, |\vec{q}| = 1, \square\left(\vec{p}, \vec{q}\right) = \frac{\pi}{6}$.

4. Naći tačku A koja leži na pravoj $a: x - y - 3 = 0, 2y + z = 0$, a od prave $b: x = y = z$ je udaljena $\sqrt{6}$.

GRUPA B

1. Riješiti matričnu jednačinu $AX^{-1}B - C = AX^{-1}$, ako je

   $$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, C = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

2. Izračunati sve vrijednosti korjena $\sqrt[4]{\frac{(1+i)^3 - (1-i)^3}{(1+i)^2 - (1-i)^2}}$.

3. Izračunati sve težišnice u trouglu ABC ako je

   $\overrightarrow{AC} = \vec{p} + \vec{q}, \overrightarrow{BC} = \vec{q} - 4\vec{p}, |\vec{p}| = 1, |\vec{q}| = 1, \square\left(\vec{p}, \vec{q}\right) = \frac{\pi}{3}$.

4. Naći ravan koja prolazi kroz pravu $a: x + 5y + z = 0, x - z + 4 = 0$ i sa ravni $x - 4y - 8z + 1 = 0$ zatvara ugao od 45^0.

GRUPA C

1. Koliko ima racionalnih članova u razvoju binoma $\left(\sqrt[3]{7}+\sqrt[4]{5}\right)^{30}$?
2. Riješiti jednačinu u skupu kompleksnih brojeva: $x^4-30x^2+289=0$.
3. Izračunati uglove trougla ABC ako je zadano:
   $\overrightarrow{BA}=\vec{p}-3\vec{q}, \overrightarrow{CA}=2\vec{p}+\vec{q}, |\vec{p}|=2, |\vec{q}|=1, \angle\left(\vec{p},\vec{q}\right)=\frac{5\pi}{6}$.
4. Naći jednačinu prave koja siječe prave $a:\frac{x+3}{2}=\frac{y-5}{3}=\frac{z}{1}$ i $b:\frac{x-10}{5}=\frac{y+7}{4}=\frac{z}{1}$ i paralelna je pravoj $c:\frac{x+2}{8}=\frac{y-1}{7}=\frac{z-3}{1}$.

GRUPA D

1. Riješiti sistem jednačina i diskutovati rješenja sistema u zavisnosti od parametra a:
$$\begin{aligned}(3-2a)x+(2-a)y+z&=a\\(2-a)x+(2-a)y+z&=1\\x+y+(2-a)z&=1\end{aligned}$$
2. Naći sve vrijednosti korjena $\sqrt[6]{-27}$.
3. Izračunati obim i površinu trougla ABC ako je
   $\overrightarrow{AC}=4\vec{p}+2\vec{q}, \overrightarrow{CB}=3\vec{q}-2\vec{p}, |\vec{p}|=1, |\vec{q}|=2, \angle\left(\vec{p},\vec{q}\right)=\frac{2\pi}{3}$.
4. Naći jednačinu ravni koja je paralelna ravni $2x+2y-z+3=0$, a od tačke $A(1,2,-1)$ ima udaljenost 3.

Neki zadaci nisu detaljno raspisani.

Grupa A

1.

$n=1 \Rightarrow \sin x = \frac{\sin^2 x}{\sin x} \Rightarrow \sin x = \sin x$ očito vrijedi

Pretpostavka: $\sin x + \sin 3x + \dots + \sin(2k-1)x = \frac{\sin^2 kx}{\sin x}$ (P)

$n=k+1 \Rightarrow \sin x + \sin 3x + \dots + \sin(2k-1)x + \sin(2k+1)x = \frac{\sin^2(k+1)x}{\sin x}$

Prema (P) lijeva strana je:

$$\frac{\sin^2 kx}{\sin x} + \sin(2k+1)x = \frac{\sin^2 kx + \sin x \cdot \sin(2k+1)x}{\sin x}$$

$$= \frac{\sin^2 kx + \sin x \cdot \sin(2kx + x)}{\sin x}$$

$$= \frac{\sin^2 kx + \sin x \cdot (\sin 2kx \cdot \cos x + \sin x \cdot \cos 2kx)}{\sin x}$$

$$= \frac{\sin^2 kx + \sin 2kx \cdot \sin x \cos x + \sin^2 x \cdot \cos 2kx}{\sin x}$$

$$= \frac{\sin^2 kx + 2\sin kx \cdot \cos kx \cdot \sin x \cdot \cos x + \sin^2 x\,(\cos^2 kx - \sin^2 kx)}{\sin x}$$

$$= \frac{\sin^2 kx + 2\sin kx \cdot \cos kx \cdot \sin x \cdot \cos x + \sin^2 x \cdot \cos^2 kx - \sin^2 x \cdot \sin^2 kx}{\sin x}$$

$$= \frac{\sin^2 kx\,(1 - \sin^2 x) + 2\sin kx \cdot \cos kx \cdot \sin x \cdot \cos x + \sin^2 x \cdot \cos^2 kx}{\sin x}$$

$$= \frac{\sin^2 kx \cdot \cos^2 x + 2\sin kx \cdot \cos kx \cdot \sin x \cdot \cos x + \sin^2 x \cdot \cos^2 kx}{\sin x}$$

$$= \frac{(\sin kx \cdot \cos x + \sin x \cdot \cos kx)^2}{\sin x}$$

$$= \frac{\sin^2(kx + x)}{\sin x} = \frac{\sin^2(k+1)x}{\sin x}.$$

2. $1+i\sqrt{3} = 2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) = 2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

$1+i = \sqrt{2}\cdot\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$\cos\frac{\pi}{12} - i\sin\frac{\pi}{12} = \cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)$

} pomnožit

$z = 2\sqrt{2}\cdot\left[\cos\left(\frac{\pi}{3}+\frac{\pi}{4}-\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{3}+\frac{\pi}{4}-\frac{\pi}{12}\right)\right]$

$= (\sqrt{2})^3\cdot\left(\cos\frac{6\pi}{12}+i\sin\frac{6\pi}{12}\right)$

$= (\sqrt{2})^3\cdot\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

$\sqrt[3]{z} = \sqrt{2}\cdot\left(\cos\frac{\frac{\pi}{2}+2k\pi}{3}+i\sin\frac{\frac{\pi}{2}+2k\pi}{3}\right),\ k=0,1,2$

$k=0 \Rightarrow w_1 = \sqrt{2}\cdot\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right) = \sqrt{2}\cdot\left(\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}\right)$

$w_1 = \frac{\sqrt{6}}{2}+i\cdot\frac{\sqrt{2}}{2}$

$k=1 \Rightarrow w_2 = \sqrt{2}\cdot\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

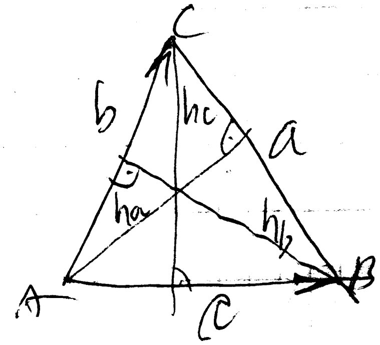
$= \sqrt{2}\cdot\left(-\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}\right) = -\frac{\sqrt{6}}{2}+i\cdot\frac{\sqrt{2}}{2}$

$k=2 \Rightarrow w_3 = \sqrt{2}\cdot\left(\cos\frac{9\pi}{6}+i\sin\frac{9\pi}{6}\right) = \sqrt{2}\cdot\left(\underbrace{\cos\frac{3\pi}{2}}_{0}+i\underbrace{\sin\frac{3\pi}{2}}_{-1}\right)$

$w_3 = -\sqrt{2}\,i$

$\sqrt[3]{z} \in \left\{\pm\frac{\sqrt{6}}{2}+i\cdot\frac{\sqrt{2}}{2},\ -\sqrt{2}\,i\right\}$

3.



$\vec{AB} = 2\vec{p}-\vec{q}$

$\vec{BC} = 3\vec{q}$

$|\vec{p}|=2,\ |\vec{q}|=1,\ \sphericalangle(\vec{p},\vec{q}) = \frac{\pi}{6}$

$$\vec{AB} \times \vec{BC} = (2\vec{p} - \vec{q}) \times 3\vec{q} = 6\,\vec{p} \times \vec{q} - \underbrace{\vec{q} \times 3\vec{q}}_{\vec{0}}$$

$$\Rightarrow P = 6 \cdot |\vec{p} \times \vec{q}| = 6 \cdot |\vec{p}| \cdot |\vec{q}| \cdot \sin \frac{\pi}{6}$$

$$P = 6 \cdot 2 \cdot 1 \cdot \frac{1}{2} \Rightarrow \boxed{P = 6}$$

$$a = |\vec{BC}| = |3\vec{q}| = 3 \cdot |\vec{q}| = 3$$

$$h_a = \frac{2P}{a} = \frac{2 \cdot 6}{3} = 4$$

$$b = |\vec{AC}| = |\vec{AB} + \vec{BC}| = |2\vec{p} - \vec{q} + 3\vec{q}| = |2\vec{p} + 2\vec{q}|$$

$$= 2 \cdot |\vec{p} + \vec{q}| = 2 \cdot \sqrt{(\vec{p} + \vec{q})^2} = 2 \cdot \sqrt{\vec{p}^2 + 2\vec{p}\vec{q} + \vec{q}^2}$$

$$= 2 \cdot \sqrt{4 + 2 \cdot 2 \cdot 1 \cdot \frac{\sqrt{3}}{2} + 1} = 2 \cdot \sqrt{5 + 2\sqrt{3}}$$

$$h_b = \frac{2P}{b} = \frac{2 \cdot 6}{2 \cdot \sqrt{5 + 2\sqrt{3}}} = \frac{6}{\sqrt{5 + 2\sqrt{3}}}$$

$$c = |\vec{AB}| = |2\vec{p} - \vec{q}| = \sqrt{(2\vec{p} - \vec{q})^2} =$$

$$= \sqrt{4\vec{p}^2 - 4\vec{p} \cdot \vec{q} + \vec{q}^2} = \sqrt{4 \cdot 4 - 4 \cdot 2 \cdot 1 \frac{\sqrt{3}}{2} + 1} = \sqrt{17 - 4\sqrt{3}}$$

$$h_c = \frac{2P}{c} = \frac{2 \cdot 6}{\sqrt{17 - 4\sqrt{3}}} = \frac{12}{\sqrt{17 - 4\sqrt{3}}}$$

$$a: \begin{cases} x - y - 3 = 0 \\ 2y + z = 0 \end{cases} \Rightarrow \begin{cases} x - 3 = y \\ y = -\frac{z}{2} \end{cases} \Rightarrow \frac{x-3}{1} = \frac{y}{1} = \frac{z}{-2} = t$$

$$\Rightarrow \begin{cases} x = t + 3 \\ y = t \\ z = -2t \end{cases}$$

Vzdialenosť točke $M(t+3, t, -2t)$ od priamky b:

$$d = \frac{|\vec{p} \times \vec{M_1M}|}{|\vec{p}|}, \quad \vec{p} = (1,1,1), \; M_1(3,0,0) \in b$$

→ vektor pravca prave b

$$\vec{M_1M} = (t+3, t, -2t)$$

$$\vec{p} \times \vec{M_1M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ t+3 & t & -2t \end{vmatrix} = (-3t, 3-3t, -3)$$

$$\sqrt{6} = \frac{\sqrt{9t^2 + (3-3t)^2 + 9}}{\sqrt{1+1+1}} \quad / \cdot \sqrt{3}$$

$$\sqrt{18} = \sqrt{9t^2 + 9 - 18t + 9t^2 + 9} \quad /^2$$

$$18 = 18t^2 - 18t + 18 \quad /:18$$

$$t^2 - t = 0$$

$$t(t-1) = 0 \Rightarrow t_1 = 0, \; t_2 = 1$$

$$t = 0 \Rightarrow A(3,0,0),$$

$$t = 1 \Rightarrow B(4,1,-2)$$

Imamo 2 tačke koje su udaljene od prave b za $\sqrt{6}$.

Grupa B MF

1. $AX^{-1}B - C = AX^{-1}$

$AX^{-1}B - AX^{-1} = C$

$AX^{-1}(B - I) = C \quad / \cdot A^{-1}$ lijevo

$X^{-1}(B-I) = A^{-1} \cdot C \quad / \cdot X$ lijevo

$B - I = X \cdot A^{-1} \cdot C \quad / \cdot C^{-1}$ desno, $/ \cdot A$ desno

$X = (B-I) \cdot C^{-1} \cdot A$

$\det C = 2 \Rightarrow C^{-1} = \frac{1}{2} C^*, \quad C^* = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

$X = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$

$X = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -2 & 5 \\ 0 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$

$X = \begin{bmatrix} \frac{1}{2} & 1 & 2 \\ 0 & 8 & 1 \\ 0 & -4 & -2 \end{bmatrix}$

2. $\frac{(1+i)^3 - (1-i)^3}{(1+i)^2 - (1-i)^2} = \frac{1 + 3i + 3i^2 + i^3 - (1 - 3i + 3i^2 - i^3)}{1 + 2i + i^2 - (1 - 2i + i^2)}$

$= \frac{3i - i + 3i - i}{2i + 2i} = \frac{4i}{4i} = 1 = \cos 0 + i \sin 0$

$z_k = \sqrt[4]{1} = \cos \frac{2k\pi}{4} + i \sin \frac{2k\pi}{4}, \quad k = 0, 1, 2, 3$

$k=0 \Rightarrow z_0 = 1$

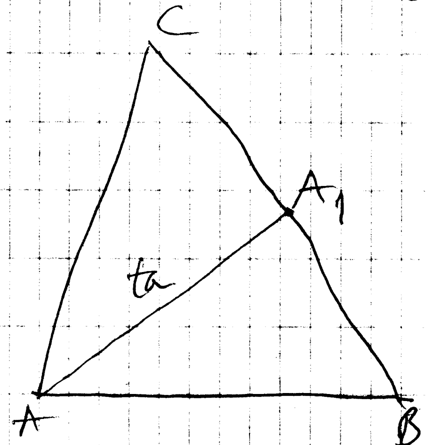
$k=1 \Rightarrow z_1 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

$k=2 \Rightarrow z_2 = \cos\pi + i\sin\pi = -1$

$k=3 \Rightarrow z_3 = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$

$\Rightarrow \sqrt[4]{z} \in \{\pm 1, \pm i\}$

3.



$\vec{AC} = \vec{p} + \vec{q}$

$\vec{BC} = \vec{q} - 4\vec{p}$

$|\vec{p}| = |\vec{q}| = 1, \quad \angle(\vec{p}, \vec{q}) = \frac{\pi}{3}$

$\vec{AB} = \vec{AC} + \vec{CB} = \vec{AC} - \vec{BC}$

$= \vec{p} + \vec{q} - (\vec{q} - 4\vec{p}) = 5\vec{p}$

$\left.\begin{array}{l}\vec{AA_1} = \vec{AB} + \vec{BA_1} \\ \vec{AA_1} = \vec{AC} + \vec{CA_1}\end{array}\right\} +$

$2\vec{AA_1} = \vec{AB} + \vec{AC} \Rightarrow \vec{AA_1} = \frac{\vec{AB} + \vec{AC}}{2}$

$\vec{AA_1} = \frac{5\vec{p} + \vec{p} + \vec{q}}{2} = \frac{6\vec{p} + \vec{q}}{2} = 3\vec{p} + \frac{1}{2}\vec{q}$

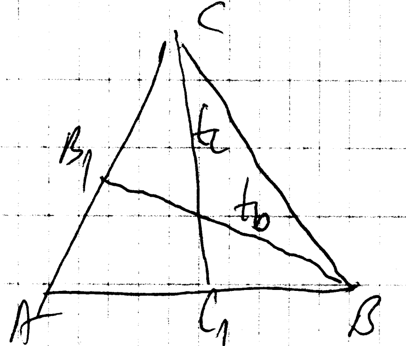
$t_a = |\vec{AA_1}| = \sqrt{(3\vec{p} + \frac{1}{2}\vec{q})^2} =$

$= \sqrt{9\vec{p}^2 + 3\vec{p}\cdot\vec{q} + \frac{1}{4}\vec{q}^2} =$

$= \sqrt{9\cdot 1 + 3\cdot 1\cdot 1\cdot\frac{1}{2} + \frac{1}{4}} = \sqrt{\frac{36 + 6 + 1}{4}} = \frac{\sqrt{43}}{2}$

$\vec{BB_1} = \frac{\vec{BA} + \vec{BC}}{2} = \frac{-5\vec{p} + \vec{q} - 4\vec{p}}{2} = \frac{-9\vec{p} + \vec{q}}{2}$

$t_b = |\vec{BB_1}| = \frac{1}{2}|\vec{q} - 9\vec{p}| = \frac{1}{2}\sqrt{(\vec{q} - 9\vec{p})^2}$



$$t_b = \frac{1}{2}\sqrt{\vec{q}^{\,2} - 2\cdot\vec{q}\cdot 9\vec{p} + 81\vec{p}^{\,2}}$$

$$= \frac{1}{2}\sqrt{1 - 2\cdot 9\cdot\frac{1}{2} + 81} = \frac{1}{2}\sqrt{73} = \frac{\sqrt{73}}{2}$$

$$\vec{CC_1} = \frac{\vec{CA}+\vec{CB}}{2} = \frac{-\vec{p}-\vec{q}-\vec{q}+4\vec{p}}{2} = \frac{-2\vec{q}+3\vec{p}}{2} = \frac{3}{2}\vec{p}-\vec{q}$$

$$t_c = \sqrt{\left(\frac{3}{2}\vec{p}-\vec{q}\right)^2} = \sqrt{\frac{9}{4} - 3\cdot\frac{1}{2} + 1} = \sqrt{\frac{9-6+4}{4}} = \frac{\sqrt{7}}{2}$$

4. Postavimo pramen ravni:

$$x+5y+z+\lambda(x-z+4)=0$$

$$x(1+\lambda) + 5y + z(1-\lambda) + 4\lambda = 0$$

$$\alpha: x-4y-8z+1=0 \Rightarrow \vec{n_\alpha} = (1,-4,-8)$$

$\vec{n} = (1+\lambda, 5, 1-\lambda)$ – za traženu ravan

$$\angle(\vec{n},\vec{n_\alpha}) = \frac{\pi}{4} \Rightarrow \cos\frac{\pi}{4} = \frac{\vec{n}\cdot\vec{n_\alpha}}{|\vec{n}|\cdot|\vec{n_\alpha}|}$$

$$\frac{\sqrt{2}}{2} = \frac{1+\lambda-20-8+8\lambda}{\sqrt{(1+\lambda)^2+25+(1-\lambda)^2}\cdot\sqrt{1+16+64}}$$

$$\frac{\sqrt{2}}{2} = \frac{9\lambda-27}{9\sqrt{2\lambda^2+27}} \quad /:9$$

$$\frac{\sqrt{2}}{2} = \frac{\lambda-3}{\sqrt{2\lambda^2+27}} \Rightarrow \sqrt{2(2\lambda^2+27)} = 2(\lambda-3) \quad /^2$$

$$4\lambda^2 + 54 = 4(\lambda - 3)^2$$

$$4\lambda^2 + 54 = 4(\lambda^2 - 6\lambda + 9)$$

$$\cancel{4\lambda^2} + 54 = \cancel{4\lambda^2} - 24\lambda + 36$$

$$24\lambda = 36 - 54$$

$$24\lambda = -18$$

$$\lambda = -\frac{18}{24} = -\frac{3}{4}$$

Tražena ravan glasi:

$$x\left(1 - \frac{3}{4}\right) + 5y + z\left(1 + \frac{3}{4}\right) - 3 = 0$$

$$x \cdot \frac{1}{4} + 5y + z \cdot \frac{7}{4} - 3 = 0 \quad / \cdot 4$$

$$\boxed{x + 20y + 7z - 12 = 0}$$

Grupa C MP

1. $\left(\sqrt[3]{7}+\sqrt[4]{5}\right)^{30}=\sum_{k=0}^{30}\binom{30}{k}\left(\sqrt[3]{7}\right)^{30-k}\cdot\left(\sqrt[4]{5}\right)^{k}$

$=\sum_{k=0}^{30}\binom{30}{k}7^{\frac{30-k}{3}}\cdot 5^{\frac{k}{4}}$

Tražimo $k\in\{0,1,\dots,30\}$ tako da

$4|k$ i $3|(30-k)$

$\Rightarrow 4|k$ i $3|k$ (jer očito $3|30$)

$\Rightarrow 12|k \Rightarrow k\in\{0,12,24\}$

Postoje 3 racionalna člana u razvoju $\left(\sqrt[3]{7}+\sqrt[4]{5}\right)^{30}$.

2. $x^4-30x^2+289=0$

$x^2=t \Rightarrow t^2-30t+289=0$

$D=30^2-4\cdot 289=900-1156=-256$

$t_{1,2}=\frac{30\pm 16i}{2}=15\pm 8i$

$x^2=15+8i \Rightarrow x_{1,2}=\sqrt{15+8i}$

$x^2=15-8i \Rightarrow x_{3,4}=\sqrt{15-8i}$

$z_1=15+8i \Rightarrow |z_1|=\sqrt{225+64}=17$

$z_1=17\cdot\left(\frac{15}{17}+\frac{8i}{17}\right) \Rightarrow \cos\varphi_1=\frac{15}{17},\ \sin\varphi_1=\frac{8}{17}$

$\sqrt{17\cdot(\cos\varphi_1+i\sin\varphi_1)}=\sqrt{17}\left(\cos\frac{\varphi_1+2k\pi}{2}+i\sin\frac{\varphi_1+2k\pi}{2}\right),\ k=0,1$

$k=0 \Rightarrow \sqrt{z_1} = \sqrt{17}\cdot\left(\cos\frac{\varphi_1}{2} + i\sin\frac{\varphi_1}{2}\right)$

$$\cos\frac{\varphi_1}{2} = \sqrt{\frac{1+\cos\varphi_1}{2}} = \sqrt{\frac{1+\frac{15}{17}}{2}} = \sqrt{\frac{\frac{32}{17}}{2}} = \sqrt{\frac{16}{17}} = \frac{4}{\sqrt{17}}$$

$$\sin\frac{\varphi_1}{2} = \sqrt{\frac{1-\cos\varphi_1}{2}} = \sqrt{\frac{1-\frac{15}{17}}{2}} = \sqrt{\frac{\frac{2}{17}}{2}} = \sqrt{\frac{1}{17}} = \frac{1}{\sqrt{17}}$$

$$\sqrt{z_1} = \sqrt{17}\cdot\left(\frac{4}{\sqrt{17}} + i\cdot\frac{1}{\sqrt{17}}\right) = 4+i$$

$k=1 \Rightarrow \sqrt{z_1} = \sqrt{17}\cdot\left(\cos\frac{\varphi_1+2\pi}{2} + i\sin\frac{\varphi_1+2\pi}{2}\right)$

$$= \sqrt{17}\left(\cos\left(\frac{\varphi_1}{2}+\pi\right) + i\sin\left(\frac{\varphi_1}{2}+\pi\right)\right)$$

$$= \sqrt{17}\cdot\left(-\cos\frac{\varphi_1}{2} - i\sin\frac{\varphi_1}{2}\right)$$

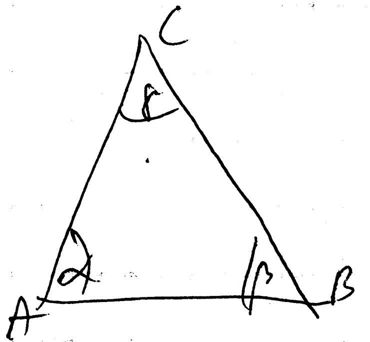
$$= \sqrt{17}\cdot\left(-\frac{4}{\sqrt{17}} - i\,\frac{1}{\sqrt{17}}\right) = -4-i$$

$\sqrt{z_1} = \pm(4+i)$

Analogno, $z_2 = 15-8i \Rightarrow \sqrt{z_2} = \pm(4-i)$

$x_{1,2,3,4} = \pm(4\pm i)$.

3.



$\alpha = \sphericalangle(\vec{AB}, \vec{AC})$

$$\cos\alpha = \frac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}|\cdot|\vec{AC}|}$$

$\vec{AB} = -\vec{BA} = -(\vec{p} - 3\vec{q}) = 3\vec{q} - \vec{p}$

$\vec{AC} = -\vec{CA} = -(2\vec{p} + \vec{q}) = -2\vec{p} - \vec{q}$

$$\vec{AB}\cdot\vec{AC} = (3\vec{q}-\vec{p})(-2\vec{p}-\vec{q}) = -6\vec{p}\vec{q} - 3\vec{q}^2 + 2\vec{p}^2 + \vec{p}\cdot\vec{q}$$
$$= -3\vec{q}^2 + 2\vec{p}^2 - 5\vec{p}\cdot\vec{q}$$
$$= -3\cdot 1 + 2\cdot 4 - 5\cdot 1\cdot 2\cdot \cos\frac{5\pi}{6}$$
$$= -3 + 8 - 5\cdot 2\cdot\left(-\frac{\sqrt{3}}{2}\right) = 5 + 5\sqrt{3}$$

$$|\vec{AB}| = \sqrt{(3\vec{q}-\vec{p})^2} = \sqrt{9\vec{q}^2 - 6\vec{p}\vec{q} + \vec{p}^2} =$$
$$= \sqrt{9\cdot 1 - 6\cdot 2\cdot 1\left(-\frac{\sqrt{3}}{2}\right) + 4} = \sqrt{13+6\sqrt{3}}$$

$$|\vec{AC}| = \sqrt{(-2\vec{p}-\vec{q})^2} = \sqrt{4\vec{p}^2 + 4\vec{p}\cdot\vec{q} + \vec{q}^2}$$
$$= \sqrt{4\cdot 4 + 4\cdot 2\cdot 1\cdot\left(-\frac{\sqrt{3}}{2}\right) + 1} = \sqrt{17 - 4\sqrt{3}}$$

$$\cos\alpha = \frac{5+5\sqrt{3}}{\sqrt{(13+6\sqrt{3})(17-4\sqrt{3})}}$$

$\beta = \angle(\vec{BA}, \vec{BC})$, $\gamma = \angle(\vec{CA}, \vec{CB})$ – analogno

4. Neka je tražena prava l.
$l \parallel c \Rightarrow \vec{p} = (8,7,1)$ – možemo odabrati kao vektor pravca date prave. Neka je $M(x,y,z)$ jedna tačka na pravoj l.
l siječe a

$$\Rightarrow \begin{vmatrix} x+3 & y-5 & z \\ 8 & 7 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 0 \quad \ldots (1)$$

l siječe b

$$\Rightarrow \begin{vmatrix} x-10 & y+7 & z \\ 8 & 7 & 1 \\ 5 & 4 & 1 \end{vmatrix} = 0 \quad \ldots (2)$$

$(1) \Rightarrow 4x - 6y + 10z + 42 = 0 \quad / :2$

$(2) \Rightarrow 3x - 3y - 3z - 51 = 0 \quad / :3$

$$\left.\begin{aligned} 2x - 3y + 5z + 21 &= 0 \\ x - y - z - 17 &= 0 \quad / \cdot 5 \end{aligned}\right\} +$$

$$7x - 8y - 64 = 0$$

Ako odaberemo $x = 0 \Rightarrow y = -8$

$$x - y - z = 17$$

$$0 + 8 - z = 17$$

$$-z = 9 \Rightarrow z = -9$$

Tačka $M(0, -8, -9) \in l$

$$\Rightarrow l: \frac{x}{8} = \frac{y+8}{7} = \frac{z+9}{1}$$

Grupa D MF

1.
$$D = \begin{vmatrix} 3-2a & 2-a & 1 \\ 2-a & 2-a & 1 \\ 1 & 1 & 2-a \end{vmatrix} \overset{I_v - II_v}{=} \begin{vmatrix} 1-a & 0 & 0 \\ 2-a & 2-a & 1 \\ 1 & 1 & 2-a \end{vmatrix}$$

$$= (1-a)\cdot\left[(2-a)^2 - 1\right] = (1-a)(1-a)(3-a) = (a-1)^2(3-a)$$

$$D_x = \begin{vmatrix} a & 2-a & 1 \\ 1 & 2-a & 1 \\ 1 & 1 & 2-a \end{vmatrix} = \begin{vmatrix} a-1 & 0 & 0 \\ 1 & 2-a & 1 \\ 1 & 1 & 2-a \end{vmatrix}$$

$$= (a-1)\cdot(1-a)(3-a) = (a-1)^2(a-3)$$

$$D_y = \begin{vmatrix} 3-2a & a & 1 \\ 2-a & 1 & 1 \\ 1 & 1 & 2-a \end{vmatrix} \underset{II_v - III_v}{\overset{I_v - III_v}{=}} \begin{vmatrix} 2-2a & a-1 & a-1 \\ 1-a & 0 & a-1 \\ 1 & 1 & 2-a \end{vmatrix}$$

$$= (a-1)\cdot(a-1)\cdot \begin{vmatrix} -2 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 2-a \end{vmatrix}$$

$$= (a-1)^2 \cdot \begin{vmatrix} -2 & 1 & 1 \\ -1 & 0 & 1 \\ 3 & 0 & 1-a \end{vmatrix} = (a-1)^2\cdot\left[-(-1+a-3)\right]$$

$$= -(a-1)^2(a-4)$$

$$D_z = \begin{vmatrix} 3-2a & 2-a & a \\ 2-a & 2-a & 1 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2-2a & 1-a & a-1 \\ 1-a & 1-a & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (a-1)^2 \begin{vmatrix} -2 & -1 & 1 \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (a-1)^2 \begin{vmatrix} -2 & -1 & 1 \\ -1 & -1 & 0 \\ 3 & 2 & 0 \end{vmatrix} = (a-1)^2$$

Diskusija:

1° $a \neq 1 \wedge a \neq 3 \Rightarrow$ sistem ima tačno jedno rješenje

$x = -1,\ y = \frac{a-4}{a-3},\ z = \frac{1}{3-a}$

2° $a = 1 \Rightarrow D = D_x = D_y = D_z = 0$

$x + y + z = 1$
$x + y + z = 1$
$x + y + z = 1$

} beskonačno mnogo rješenja

$y = \alpha,\ z = \beta \Rightarrow x = 1 - \alpha - \beta$

Rješenja $(1-\alpha-\beta, \alpha, \beta)$

3° $a = 3 \Rightarrow$ nema rješenja $(D = 0, D_z \neq 0)$

2. $-27 = 27(\cos\pi + i\sin\pi)$

$z_k = \sqrt[6]{27}\left(\cos\frac{\pi + 2k\pi}{6} + i\sin\frac{\pi + 2k\pi}{6}\right),\ k = 0, 1, \ldots, 5$

$\sqrt[6]{27} = \sqrt[6]{3^3} = \sqrt{3}$

$k = 0 \Rightarrow z_0 = \sqrt{3}\cdot\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = \frac{3}{2} + \frac{i\sqrt{3}}{2}$

$k = 1 \Rightarrow z_1 = \sqrt{3}\cdot i$

$k = 2 \Rightarrow z_2 = \sqrt{3}\cdot\left(-\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = -\frac{3}{2} + i\cdot\frac{\sqrt{3}}{2}$

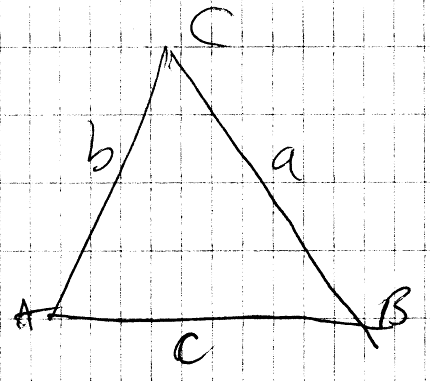
$k = 3 \Rightarrow z_3 = \sqrt{3}\left(-\frac{\sqrt{3}}{2} - i\cdot\frac{1}{2}\right) = -\frac{3}{2} - \frac{i\sqrt{3}}{2}$

$k = 4 \Rightarrow z_4 = -\sqrt{3}i$

$k = 5 \Rightarrow z_5 = \sqrt{3}\cdot\left(\frac{\sqrt{3}}{2} - i\cdot\frac{1}{2}\right) = \frac{3}{2} - \frac{i}{2}\sqrt{3}$

$\sqrt[6]{-27} \in \left\{\pm\sqrt{3}i, \pm\frac{3}{2} \pm \frac{i\sqrt{3}}{2}\right\}$

3.



$O = a + b + c$

$a = |\vec{BC}| = |\vec{CB}| = |3\vec{q} - 2\vec{p}|$

$= \sqrt{(3\vec{q} - 2\vec{p})^2}$

$= \sqrt{9\vec{q}^2 - 12\vec{p}\cdot\vec{q} + 4\vec{p}^2}$

$= \sqrt{9 \cdot 4 - 12 \cdot 1 \cdot 2 \cdot (-\frac{1}{2}) + 4 \cdot 1} = \sqrt{36 + 12 + 4} = \sqrt{52} = 2\sqrt{13}$

$b = |\vec{AC}| = \sqrt{(4\vec{p} + 2\vec{q})^2} = \sqrt{16\vec{p}^2 + 16\vec{p}\vec{q} + 4\vec{q}^2}$

$= \sqrt{16 \cdot 1 + 16 \cdot 1 \cdot 2 \cdot (-\frac{1}{2}) + 4 \cdot 4}$

$= \sqrt{16 - 16 + 16} = \sqrt{16} = 4$

$c = |\vec{AB}|, \quad \vec{AB} = \vec{AC} + \vec{CB} = 2\vec{p} + 5\vec{q}$

$c = \sqrt{4\vec{p}^2 + 20\vec{p}\vec{q} + 25\vec{q}^2} = \sqrt{4 \cdot 1 + 20 \cdot 1 \cdot 2 \cdot (-\frac{1}{2}) + 25 \cdot 4}$

$= \sqrt{4 - 20 + 100} = \sqrt{84} = \sqrt{4 \cdot 21} = 2\sqrt{21}$

$O = 2\sqrt{13} + 4 + 2\sqrt{21} = 2(\sqrt{13} + 2 + \sqrt{21})$

$P = \frac{1}{2}|\vec{AC} \times \vec{CB}| = \frac{1}{2}|(4\vec{p} + 2\vec{q}) \times (3\vec{q} - 2\vec{p})| =$

$= \frac{1}{2}|12\vec{p} \times \vec{q} - 4\vec{q} \times \vec{p}| = \frac{1}{2}|12\vec{p} \times \vec{q} + 4\vec{p} \times \vec{q}| =$

$= \frac{1}{2} \cdot 16 \cdot |\vec{p} \times \vec{q}| = 8 \cdot |\vec{p}| \cdot |\vec{q}| \cdot \sin\frac{2\pi}{3} = 8 \cdot 2 \cdot 1 \cdot \frac{\sqrt{3}}{2}$

$P = 8\sqrt{3}$

4. $\vec{n} = (2, 2, -1)$

Tražena ravan glasi:

$\alpha: 2x + 2y - z + D = 0$

Udaljenost tačke $A(1, 2, -1)$ od ravni α je

$$d = \frac{|2 \cdot 1 + 2 \cdot 2 - (-1) + D|}{\sqrt{4 + 4 + 1}} = 3$$

$$\frac{|7 + D|}{3} = 3$$

$|7 + D| = 9 \Rightarrow 7 + D = \pm 9 \Rightarrow D = \pm 9 - 7$

$D_1 = 9 - 7 = 2$

$D_2 = -9 - 7 = -16$

Imamo 2 tražene ravni:

$2x + 2y - z + 2 = 0$ i

$2x + 2y - z - 16 = 0$